Exercise 2.3 — Using the binomial probability function

Q1 **a)** Use the binomial probability function with $n = 10$ and $p = 0.14$.

(i) $$\mathrm{P}(X = 2) = \binom{10}{2} \times 0.14^2 \times (1 - 0.14)^{10-2}$$

$$= \frac{10!}{2!8!} \times 0.14^2 \times 0.86^8 = 0.264 \text{ (to 3 s.f.)}$$

(ii) $$\mathrm{P}(X = 4) = \binom{10}{4} \times 0.14^4 \times (1 - 0.14)^{10-4}$$

$$= \frac{10!}{4!6!} \times 0.14^4 \times 0.86^6 = 0.0326 \text{ (to 3 s.f.)}$$

(iii) $$\mathrm{P}(X = 5) = \binom{10}{5} \times 0.14^5 \times (1 - 0.14)^{10-5}$$

$$= \frac{10!}{5!5!} \times 0.14^5 \times 0.86^5 = 0.00638 \text{ (to 3 s.f.)}$$

b) Use the binomial probability function with $n = 8$ and $p = 0.27$.

(i) $$\mathrm{P}(X = 3) = \binom{8}{3} \times 0.27^3 \times (1 - 0.27)^{8-3}$$

$$= \frac{8!}{3!5!} \times 0.27^3 \times 0.73^5 = 0.229 \text{ (to 3 s.f.)}$$

(ii) $$\mathrm{P}(X = 5) = \binom{8}{5} \times 0.27^5 \times (1 - 0.27)^{8-5}$$

$$= \frac{8!}{5!3!} \times 0.27^5 \times 0.73^3 = 0.0313 \text{ (to 3 s.f.)}$$

(iii) $$\mathrm{P}(X = 7) = \binom{8}{7} \times 0.27^7 \times (1 - 0.27)^{8-7}$$

$$= \frac{8!}{7!1!} \times 0.27^7 \times 0.73^1 = 0.000611 \text{ (to 3 s.f.)}$$

Q2 **a)** Use the binomial probability function with $n = 20$ and $p = 0.16$.

(i) $P(X < 2) = P(X = 0) + P(X = 1)$

$$= \frac{20!}{0!20!} \times 0.16^0 \times 0.84^{20} + \frac{20!}{1!19!} \times 0.16^1 \times 0.84^{19}$$

$= 0.03059... + 0.11653... = 0.147$ (to 3 s.f.)

(ii) $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$$= 0.03059... + 0.11653... + \frac{20!}{2!18!} \times 0.16^2 \times 0.84^{18}$$

$$+ \frac{20!}{3!17!} \times 0.16^3 \times 0.84^{17}$$

$= 0.03059... + 0.11653... + 0.21087... + 0.24099...$

$= 0.599$ (to 3 s.f.)

(iii) $P(1 < X \leq 4) = P(X = 2) + P(X = 3) + P(X = 4)$

$$= 0.21087... + 0.24099... + \frac{20!}{4!16!} \times 0.16^4 \times 0.84^{16}$$

$= 0.21087... + 0.24099... + 0.19509...$

$= 0.647$ (to 3 s.f.)

b) Use the binomial probability function with $n = 30$ and $p = 0.88$.

(i) $P(X > 28) = P(X = 29) + P(X = 30)$

$$= \frac{30!}{29!1!} \times 0.88^{29} \times 0.12^1 + \frac{30!}{30!0!} \times 0.88^{30} \times 0.12^0$$

$= 0.088369... + 0.021601...$

$= 0.110$ (to 3 s.f.)

(ii) $P(25 < X < 28) = P(X = 26) + P(X = 27)$

$$= \frac{30!}{26!4!} \times 0.88^{26} \times 0.12^4 + \frac{30!}{27!3!} \times 0.88^{27} \times 0.12^3$$

$= 0.204693... + 0.222383...$

$= 0.427$ (to 3 s.f.)

(iii) $P(X \geq 27) = P(X = 27) + P(X = 28)$
$+ P(X = 29) + P(X = 30)$

$$= 0.222383... + \frac{30!}{28!2!} \times 0.88^{28} \times 0.12^2$$

$+ 0.088369... + 0.021601...$

$= 0.222383... + 0.174729... + 0.088369...$
$+ 0.021601...$

$= 0.507$ (to 3 s.f.)

Q3 **a)** Use the binomial probability function with $n = 5$ and $p = \frac{1}{2}$.

(i) $P(X \leq 4) = 1 - P(X > 4) = 1 - P(X = 5)$

$= 1 - \frac{5!}{5!0!} \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^0 = 1 - 0.03125$

$= 0.969$ (to 3 s.f.)

(ii) $P(X > 1) = 1 - P(X \leq 1) = 1 - P(X = 0) - P(X = 1)$

$= 1 - \frac{5!}{0!5!} \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^5 - \frac{5!}{1!4!} \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^4$

$= 1 - 0.03125 - 0.15625 = 0.813$ (to 3 s.f.)

(iii) $P(1 \leq X \leq 4) = 1 - P(X = 0) - P(X = 5)$

$= 1 - \frac{5!}{0!5!} \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^5 - \frac{5!}{5!0!} \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^0$

$= 1 - 0.03125 - 0.03125 = 0.938$ (to 3 s.f.)

b) Use the binomial probability function with $n = 8$ and $p = \frac{2}{3}$.

(i) $P(X < 7) = 1 - P(X \geq 7) = 1 - P(X = 7) - P(X = 8)$

$= 1 - \frac{8!}{7!1!} \times \left(\frac{2}{3}\right)^7 \times \left(\frac{1}{3}\right)^1 - \frac{8!}{8!0!} \times \left(\frac{2}{3}\right)^8 \times \left(\frac{1}{3}\right)^0$

$= 1 - 0.156073... - 0.039018... = 0.805$ (to 3 s.f.)

(ii) $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1)$

$= 1 - \frac{8!}{0!8!} \times \left(\frac{2}{3}\right)^0 \times \left(\frac{1}{3}\right)^8 - \frac{8!}{1!7!} \times \left(\frac{2}{3}\right)^1 \times \left(\frac{1}{3}\right)^7$

$= 1 - 0.00015241... - 0.00243865...$

$= 0.997$ (to 3 s.f.)

(iii) $P(0 \leq X \leq 8) = 1$

This must be 1, as X can only take values from 0 to 8.

Q4 $n = 5$ and $p = P(\text{roll a six}) = \frac{1}{6}$, so

$P(2 \text{ sixes}) = \binom{5}{2} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^3 = 0.161$ (to 3 s.f.)

Q5 **a)** There are 12 answers, which are either 'correct' or 'incorrect'. The student guesses at random so the questions are answered independently of each other and the probability of a correct answer is $\frac{1}{3}$.

So $X \sim B(12, \frac{1}{3})$.

b) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{12!}{0!12!} \times \left(\frac{1}{3}\right)^0 \times \left(\frac{2}{3}\right)^{12} + \frac{12!}{1!11!} \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^{11}$$
$$+ \frac{12!}{2!10!} \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^{10}$$
$$= 0.00770... + 0.04624... + 0.12717...$$
$$= 0.181 \text{ (to 3 s.f.)}$$

Q6 Let X represent the number of defective items.
Then $X \sim B(15, 0.05)$, and you need to find $P(1 \leq X \leq 3)$.
$P(1 \leq X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3)$

$$= \frac{15!}{1!14!} \times 0.05^1 \times 0.95^{14} + \frac{15!}{2!13!} \times 0.05^2 \times 0.95^{13}$$
$$+ \frac{15!}{3!12!} \times 0.05^3 \times 0.95^{12}$$
$$= 0.36575... + 0.13475... + 0.03073... = 0.531 \text{ (to 3 s.f.)}$$

Q7 **a)** Let the random variable X represent the number of 'treble-20's the player gets in a set of 3 darts.
Then $X \sim B(3, 0.75)$.
$P(X \geq 2) = P(X = 2) + P(X = 3)$

$$P(X = 2) = \binom{3}{2} \times 0.75^2 \times (1 - 0.75) = 0.421875$$
$$P(X = 3) = \binom{3}{3} \times 0.75^3 \times (1 - 0.75)^0 = 0.421875$$

So $P(X \geq 2) = 0.421875 + 0.421875$
$= 0.84375 = 0.844$ (to 3 s.f.)

b) Now let X represent the number of 'treble-20's the player scores with 30 darts. Then $X \sim B(30, 0.75)$.
You need to find $P(X \geq 26)$.
$P(X \geq 26) = P(X = 26) + P(X = 27) + P(X = 28)$
$+ P(X = 29) + P(X = 30)$
$= 0.06042... + 0.02685... + 0.00863...$
$+ 0.00178... + 0.00017... = 0.0979$ (to 3 s.f.)

Exercise 3.1 — Using a calculator to find probabilities

Q1 **a)** $P(X \le 2) = 0.526$ (3 s.f.)

b) $P(X \le 6) = 0.996$ (3 s.f.)

c) $P(X \le 9) = 1.00$ (3 s.f.)

d) $P(X < 5) = P(X \le 4) = 0.922$ (3 s.f.)

e) $P(X < 4) = P(X \le 3) = 0.776$ (3 s.f.)

f) $P(X < 6) = P(X \le 5) = 0.980$ (3 s.f.)

Q2 **a)** $P(X > 3) = 1 - P(X \le 3) = 1 - 0.0905... = 0.909$ (3 s.f.)

b) $P(X > 6) = 1 - P(X \le 6) = 1 - 0.6098... = 0.390$ (3 s.f.)

c) $P(X > 10) = 1 - P(X \le 10) = 1 - 0.990652...$
$= 0.00935$ (3 s.f.)

d) $P(X \ge 5) = 1 - P(X < 5) = 1 - P(X \le 4)$
$= 1 - 0.2172... = 0.783$ (3 s.f.)

e) $P(X \ge 3) = 1 - P(X < 3) = 1 - P(X \le 2)$
$= 1 - 0.0271... = 0.973$ (3 s.f.)

f) $P(X \ge 13) = 1 - P(X < 13) = 1 - P(X \le 12)$
$= 1 - 0.9997210... = 0.000279$ (3 s.f.)

Q3 **a)** $P(X = 7) = 0.184$ (3 s.f.)

If your calculator doesn't have a binomial pdf, you can either work these out using the binomial probability formula, or calculate $P(X \le 7) - P(X \le 6)$ with the binomial cdf.

b) $P(X = 12) = 0.0136$ (3 s.f.)

c) $P(2 < X \le 4) = P(X \le 4) - P(X \le 2)$
$= 0.11819... - 0.01211... = 0.106$ (3 s.f.)

d) $P(10 < X \le 15) = P(X \le 15) - P(X \le 10)$
$= 0.99995... - 0.94683...$
$= 0.0531$ (3 s.f.)

e) $P(7 \le X \le 10) = P(X \le 10) - P(X \le 6)$
$= 0.94683... - 0.41662... = 0.530$ (3 s.f.)

f) $P(3 \le X < 11) = P(X \le 10) - P(X \le 2)$
$= 0.94683... - 0.01211... = 0.935$ (3 s.f.)

Q4 **a)** $P(X \ge 17) = 1 - P(X < 17) = 1 - P(X \le 16)$
$= 1 - 0.04677... = 0.953$ (3 s.f.)

b) $P(X \ge 20) = 1 - P(X < 20) = 1 - P(X \le 19)$
$= 1 - 0.38331... = 0.617$ (3 s.f.)

c) $P(X > 14) = 1 - P(X \le 14)$
$= 1 - 0.00555... = 0.994$ (3 s.f.)

d) $P(X = 21) = 0.187$ (3 s.f.)

e) $P(14 \le X < 17) = P(X \le 16) - P(X \le 13)$
$= 0.04677... - 0.00154... = 0.0452$ (3 s.f.)

f) $P(12 < X \le 18) = P(X \le 18) - P(X \le 12)$
$= 0.21996... - 0.00036... = 0.220$ (3 s.f.)

Q5 Let X represent the number of children with green eyes.
Then $X \sim B(30, 0.18)$.
$P(X < 10) = P(X \le 9) = 0.96768... = 0.968$ (3 s.f.)

Q6 Let X represent the number of faulty items.
Then $X \sim B(25, 0.05)$.
$P(X < 6) = P(X \le 5) = 0.99878... = 0.999$ (3 s.f.)